HEAT AND MASS TRANSFER DUE TO RAPID CONVECTION IN CAPILLARY-POROUS MATERIALS

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A differential equation is given for the heat and mass transfer accompanying the passage of a rapid stream of air through a capillary-porous substance.

There are differential equations for molar–molecular mass and heat transfer in capillary-porous substances [1-3].

It was assumed in the derivation of these equations that the vapor–air mixture moves slowly through the pores and, hence, is in thermal and molecular equilibrium with the liquid and "skeleton."

There are cases in practice where air is blown through a porous material to accelerate heat and mass transfer. Moisture is transported mainly in the form of vapor, which is entrained by the air flow and forms a binary mixture with it. The temperature of the vapor–air mixture in this case may differ from the temperature of the "skeleton" and the liquid-phase moisture in contact with it.

An example of such a process is the drying of raw cotton by blowing air through it.

A step towards the obtention of an analytical expression connecting the physical characteristics of the effects occurring in such processes is to derive an appropriate differential equation.

We consider a capillary-porous or colloidal capillary-porous substance with a lyophilic "skeleton."

The mass-transfer equations can be written in the form [3]

$$\frac{\partial(\rho_0 u_i)}{\partial\tau} = -\operatorname{div}(\mathbf{j}_{mi} + \mathbf{j}_{ki}) + I_i \quad (i = 1, 2, 3). \tag{1}$$

In the case of a rapid flow of air we can assume that:

$$\begin{gathered} \mathbf{j}_{mi} = 0, \\ \mathbf{j}_{ki} = 0, \\ \operatorname{div} j_{k3} = 0, \\ I_1 = -I_2, \\ I_3 = 0. \end{gathered} \tag{2}$$

Then (1) can be written in the form

$$\begin{gathered} \rho_0 \frac{\partial u_1}{\partial\tau} = -I_2, \\ \rho_0 \frac{\partial u_2}{\partial\tau} = 0, \\ \rho_0 \frac{\partial u_3}{\partial\tau} = 0. \end{gathered} \tag{3}$$

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Using (2) and (3) we can put the differential heat-transfer equation

$$\frac{\partial}{\partial \tau}\left(h_0\rho_0 + \sum_{i=1}^{3} h_i\rho_0 u_i\right) = -\operatorname{div}\left[\mathbf{j}_q + \sum_{i=1}^{3}(\mathbf{j}_{mi} + \mathbf{j}_{ki})\, h_i\right] \tag{4}$$

for a material like raw cotton, which has low thermal conductivity, in the form

$$C\rho_0 \frac{\partial t}{\partial \tau} + c_V u_V \rho_0 \frac{\partial \theta}{\partial \tau} = (c_1 t - c_2\theta)\, I_2. \tag{5}$$

Here $c = c_0 + c_1u_1$ is the specific heat of the moist material; $c_Vu_V = c_2u_2 + c_3u_3$ is the specific heat of the vapor–air mixture; the substantial derivative is

$$\frac{d\theta}{d\tau} = \frac{\partial \theta}{\partial \tau} + \mathbf{w} \operatorname{grad} \theta.$$

If we introduce a bulk heat-transfer coefficient α_V and a specific latent heat of vaporization, the fact that the heat transferred from the vapor–air mixture goes towards heating of the moist material, phase transition, and heating of the vapor, can be written in the form

$$\alpha_v(\theta - t) = C\rho_0 \frac{\partial t}{\partial \tau} + [r + c_2(\theta - t)]\, I_2. \tag{6}$$

The second term in the square brackets can be neglected in view of its smallness in comparison with the first term and then

$$\theta = t + \frac{1}{\alpha}\left(C\rho_0 \frac{\partial t}{\partial \tau} + rI_2\right), \tag{7}$$

$$\frac{d\theta}{d\tau} = \frac{C\rho_0}{\alpha_v} \frac{\partial^2 t}{\partial \tau^2} + \mathbf{w} \operatorname{grad}\left(\frac{C\rho_0}{\alpha_v}\frac{\partial t}{\partial \tau} + \frac{r}{\alpha_v} I_2 + t\right) + \frac{\partial t}{\partial \tau}. \tag{8}$$

Equation (5) for the one-dimensional case is written in the form

$$\frac{\partial^2 t}{\partial \tau^2} + w_x \frac{\partial^2 t}{\partial \tau \partial x} + \frac{\alpha_v}{\rho_0}\left(\frac{1}{c_V u_V} + \frac{1}{C}\right)\frac{\partial t}{\partial \tau} + w \frac{\alpha_v}{\rho_0 C}\frac{\partial t}{\partial x} - \frac{r}{C c_V u_V}\frac{\alpha_v}{\rho_0}\frac{\partial u_1}{\partial \tau} = 0. \tag{9}$$

The boundary and initial conditions are determined by the temperature and moisture content of the vapor–air mixture before entry into the layer and the initial temperature and moisture content of the porous material.

Introducing the dimensionless quantities

$$t^* = \frac{t - t_i}{\theta_i - t_i}, \quad u^* = \frac{u_i - u}{u_i}, \quad X = \frac{x}{R}, \tag{10}$$

where t_i, θ_i, and u_i are the initial temperatures and moisture content, we can put Eq. (9) in the dimensionless form

$$\frac{\partial^2 t^*}{\partial \mathrm{F}^2\mathrm{o}} + \mathrm{Pe}\frac{\partial^2 t^*}{\partial \mathrm{Fo}\,\partial X} + \mathrm{Nu}_v\left[(1 + \mathrm{K}_c)\frac{\partial t^*}{\partial \mathrm{Fo}} + \mathrm{PeK}_c\frac{\partial t^*}{\partial X} + \mathrm{Ko}\frac{\partial u^*}{\partial \mathrm{Fo}}\right] = 0. \tag{11}$$

Here $\mathrm{Nu}_V = \alpha_V R^2/\lambda_V$ – the bulk thermal Nusselt number – characterizes the increase in heat transfer between the vapor–air mixture moving through the pores of the "skeleton" over that due to heat conduction alone in a stationary medium; $K_C = c_V u_V / C$ is a parametric criterion. The other criteria are denoted by the usual symbols.

NOTATION

ρ_0 is the density of the "skeleton";
u_i is the specific mass content of i-th substance;
j_{mi} is the specific molecular flow of i-th substance;
j_{ki} is the convective flow of i-th substance;
I_i is the strength of source of i-th substance;
h_i is the specific enthalpy of i-th substance;

j_q is the heat flow according to the Fourier law;
c_i is the specific heat;
t is the averaged temperature of the "skeleton"–liquid system;
θ is the temperature of the vapor–air system;
λ is the thermal conductivity;
w is the velocity of the vapor–air mixture.

Subscripts

0 denotes the "skeleton";
1 denotes the liquid;
2 denotes the vapor;
3 denotes the air;
v denotes the vapor–air mixture.

LITERATURE CITED

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